

# Entropy, Evolution and Open Systems

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## Abstract

It is commonly argued that the spectacular increase in order which has occurred on Earth is consistent with the second law of thermodynamics because the Earth is not an isolated system, and anything can happen in a non-isolated system as long as the entropy increases outside the system compensate the entropy decreases inside the system. However, if we define “X-entropy” to be the entropy associated with any diffusing component X (for example, X might be heat), and, since entropy measures disorder, “X-order” to be the negative of X-entropy, a closer look at the equations for entropy change shows that they not only say that the X-order cannot increase in an isolated system, but that they also say that in a non-isolated system the X-order cannot increase faster than it is imported through the boundary. Thus the equations for entropy change do not support the illogical “compensation” idea; instead, they illustrate the tautology that “if an increase in order is extremely improbable when a system is isolated, it is still extremely improbable when the system is open, unless something is entering (or leaving) which makes it not extremely improbable.” Thus unless we are willing to argue that the influx of solar energy into the Earth makes the appearance of spaceships, computers and the Internet *not* extremely improbable, we have to conclude that at least the basic principle behind the second law has in fact been violated here.

**Key words:** Entropy, Second Law of Thermodynamics

## 1. Compensation

It is probably fair to say that the majority view of science today holds that physics explains all of chemistry, chemistry explains all of biology, and biology completely explains the human mind; thus, physics alone explains the human mind, and all it does.

In fact, since there are only four known forces of physics (the gravitational, electromagnetic and strong and weak nuclear forces), this means that these four forces must explain everything that has happened on Earth, according to this majority view. For example, Peter Urone, in *College Physics* [1], writes “One of the most remarkable simplifications in physics is that only four distinct forces account for all known phenomena.”

In my 2000 *Mathematical Intelligencer* article, “A Mathematician’s View of Evolution” [2], I argued against this view, asserting that the increase in order which has occurred on Earth seems to violate the underlying principle behind the second law of thermodynamics, in a spectacular way. I wrote:

I imagine visiting the Earth when it was young and returning now to find highways with automobiles on them, airports with jet airplanes, and tall buildings full of complicated equipment, such as televisions, telephones and computers. Then I imagine the construction of a gigantic computer model which starts with the initial conditions on Earth 4 billion years ago and tries to simulate the effects that the four known forces of physics would have on every atom and every subatomic particle on our planet. If we ran such a simulation out to the present day, would it predict that the basic forces of Nature would reorganize the basic particles of Nature into libraries full of encyclopedias, science texts and novels, nuclear power plants, aircraft carriers with supersonic jets parked on deck, and computers connected to laser printers, CRTs and keyboards? If we graphically displayed the positions of the atoms at the end of the simulation, would we find that cars and trucks had formed, or that supercomputers had arisen? Certainly we would not, and I do not believe that adding sunlight to the model would help much.

Anyone who has made such an argument is familiar with the standard reply: the Earth is not an isolated system, it receives energy from the sun, and entropy can decrease in a non-isolated system, as long as it is “compensated” somehow by a comparable or greater increase outside the system. For example, Isaac Asimov, in the *Smithsonian* journal [3] recognizes the apparent problem:

You can argue, of course, that the phenomenon of life may be an exception [to the second law]. Life on earth has steadily grown more complex, more versatile, more elaborate, more orderly, over the billions of years of the planet’s existence. From no life at all, living molecules were developed, then living cells, then living conglomerates of cells, worms, vertebrates, mammals, finally Man. And in Man is a three-pound brain which, as far as we know, is the most complex and orderly arrangement of matter in the universe. How could the human brain develop out of the primeval slime? How could that vast increase in order (and therefore that vast decrease in entropy) have taken place?

But Asimov concludes that there is no conflict with the second law here, because

Remove the sun, and the human brain would not have developed. ... And in the billions of years that it took for the human brain to develop, the increase in

entropy that took place in the sun was far greater; far, far greater than the decrease that is represented by the evolution required to develop the human brain.

Similarly, Peter Urone, in *College Physics* [1], writes:

Some people misuse the second law of thermodynamics, stated in terms of entropy, to say that the existence and evolution of life violate the law and thus require divine intervention. ... It is true that the evolution of life from inert matter to its present forms represents a large decrease in entropy for living systems. But it is always possible for the entropy of one part of the universe to decrease, provided the total change in entropy of the universe increases.

Some other authors appear to feel a little silly suggesting that increases in entropy anywhere in the universe could compensate for decreases on Earth, so they are careful to explain that this “compensation” only works locally; for example in *Order and Chaos* [4], the authors write:

In a certain sense the development of civilization may appear contradictory to the second law. ... Even though society can effect local reductions in entropy, the general and universal trend of entropy increase easily swamps the anomalous but important efforts of civilized man. Each localized, man-made or machine-made entropy decrease is accompanied by a greater increase in entropy of the surroundings, thereby maintaining the required increase in total entropy.

## 2. The Equations of Entropy Change

Of course the whole idea of compensation, whether by distant or nearby events, makes no sense logically: an extremely improbable event is not rendered less improbable simply by the occurrence of “compensating” events elsewhere. According to this reasoning, the second law does not prevent scrap metal from reorganizing itself into a computer in one room, as long as two computers in the next room are rusting into scrap metal — and the door is open. (Or the thermal entropy in the next room is increasing, though I am not sure how fast it has to increase to compensate computer construction!<sup>1</sup>)

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<sup>1</sup>Daniel Styer, however, in an *American Journal of Physics* article [5], apparently *has* figured out how fast thermal entropy needs to increase to compensate evolution. Assuming that “each individual organism is 1000 times more improbable than the corresponding individual was 100 years ago” (a “very generous” assumption) and using a generous estimate for the number of organisms on Earth, he calculates that the rate of decrease of entropy due to evolution is very small, only about 302 Joules

To understand where this argument comes from, we need to look at the equations for entropy change, as given in Appendix D of my 2005 John Wiley book [6], and previously in my 2001 *Mathematical Intelligencer* article [7], “Can ANYTHING Happen in an Open System?”

Consider the diffusion (conduction) of heat in a solid,  $R$ , with absolute temperature distribution  $U(x, y, z, t)$ . The first law of thermodynamics (conservation of energy) requires that

$$Q_t = -\nabla \cdot \mathbf{J} \quad (1)$$

where  $Q$  is the heat energy density ( $Q_t = c_p U_t$ ) and  $\mathbf{J}$  is the heat flux vector. The second law requires that the flux be in a direction in which the temperature is decreasing, i.e.

$$\mathbf{J} \cdot \nabla U \leq 0 \quad (2)$$

Equation 2 simply says that heat flows from hot to cold regions — because the laws of probability favor a more uniform distribution of heat energy.

“Thermal entropy” is a quantity that is used to measure randomness in the distribution of heat. The rate of change of thermal entropy,  $S$ , is given by the usual definition as

$$S_t = \iiint_R Q_t / U \, dV \quad (3)$$

Using (3) and the first law (1), after doing a (multidimensional) integration by parts, we get

$$S_t = \iiint_R -(\mathbf{J} \cdot \nabla U) / U^2 \, dV - \iint_{\partial R} (\mathbf{J} \cdot \mathbf{n}) / U \, dA \quad (4)$$

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per degree Kelvin per second! He concludes, “Presumably the entropy of the Earth’s biosphere is indeed decreasing by a tiny amount due to evolution and the entropy of the cosmic microwave background is increasing by an even greater amount to compensate for that decrease.” It should be noted that if one is dealt a given poker hand, then replaces some cards, according to Styer we can compute the resulting entropy decrease in the universe, in units of Joules per degree Kelvin (!), as  $k_B \log(N)$ , where  $k_B$  is the Boltzmann constant, if the new hand is  $N$  times more improbable than the first. It should also be noted that if organisms become 1000 times more improbable every century, that would imply that organisms today are, on the average, about  $10^{30000000}$  times “more improbable” than organisms a billion years ago, but, according to Styer, there is no conflict with the second law as long as something (anything, apparently!) is happening outside the Earth which, if reversed, would be even more improbable.

where  $\mathbf{n}$  is the outward unit normal on the boundary  $\partial R$ . From the second law (2), we see that the volume integral is nonnegative, and so

$$S_t \geq - \iint_{\partial R} (\mathbf{J} \cdot \mathbf{n})/U \, dA \quad (5)$$

From (5) it follows that  $S_t \geq 0$  in an isolated system, where there is no heat flux through the boundary ( $\mathbf{J} \cdot \mathbf{n} = 0$ ). Hence, in an isolated system, the entropy can never decrease. Since thermal entropy measures randomness (disorder) in the distribution of heat, its opposite (negative) can be referred to as “thermal order,” and we can say that the thermal order can never increase in an isolated system.

Since thermal entropy is quantifiable, the application of the second law to thermal entropy is commonly used as the model problem on which our thinking about the other, less quantifiable, applications is based. The fact that thermal entropy cannot decrease in an isolated system, but can decrease in a non-isolated system, was used to conclude that, in other applications, any entropy decrease in a non-isolated system is possible as long as it is compensated somehow by entropy increases outside this system, so that the total “entropy” (as though there were only one type) in the universe, or any other isolated system containing this system, still increases.

However, there is really nothing special about “thermal” entropy. Heat conduction is just diffusion of heat, and we can define an “X-entropy” (and an X-order = -X-entropy), to measure the randomness in the distribution of any other substance X that diffuses; for example, we can let  $U(x, y, z, t)$  represent the concentration of carbon diffusing in a solid, and use equation (3) again to define this entropy ( $c_p = 1$  now, so  $Q_t = U_t$ ), and repeat the analysis leading to equation (5), which now says that the “carbon order” cannot increase in an isolated system.<sup>2</sup>

Furthermore, equation (5) does not simply say that the X-entropy cannot decrease in an isolated system; it also says that in a non-isolated system, the X-entropy cannot decrease faster than it is exported through the boundary, because the boundary integral there represents the rate at which X-entropy is exported across the boundary. To see this, notice that without the denominator  $U$ , the integral in (3) represents the rate of change of total X (energy, if X=heat) in the system; with the denominator it represents the rate of change of X-entropy. Without the denominator  $U$ , the boundary integral in (5) represents the rate at

<sup>2</sup>“Entropy” sounds much more scientific than “order,” but note that in this paper, “order” is simply defined as the opposite of “entropy.” Where entropy is quantifiable, such as here, order is equally quantifiable. Physics textbooks also often use the term “entropy” in a less precise sense, to describe the increase in disorder associated with, for example, a plate breaking or a bomb exploding (e.g., [8], p 651). In such applications, “order” is equally difficult to quantify!

which  $X$  (energy, if  $X$ =heat) is exported through the boundary; with the denominator therefore it must represent the rate at which  $X$ -entropy is exported. Although I am certainly not the first to recognize that the boundary integral has this interpretation (see [9], p. 202)<sup>3</sup>, this has been noticed by relatively few people, no doubt because usually the special case of isotropic heat conduction or diffusion is assumed, in which case  $J = -K\nabla U$ , and then the numerator in the boundary integral is written as  $-K\partial U/\partial n$ , and in this form it is not obvious that anything is being imported or exported, only that in an isolated system, the boundary integral is zero. Furthermore, entropy as defined by (3) seems to be a rather abstract quantity, and it is hard to visualize what it means to import or export entropy.

Stated in terms of order, equation (5) says that the  $X$ -order in a non-isolated system cannot increase faster than it is imported through the boundary. According to (4), the  $X$ -order in a system can decrease in two different ways: it can be converted to disorder (first integral term) or it can be exported through the boundary (boundary integral term). It can increase in only one way: by importation through the boundary.

### 3. A Tautology

The second law of thermodynamics is all about probability; it uses probability at the microscopic level to predict macroscopic change.<sup>4</sup> Carbon distributes itself more and more uniformly in an isolated solid because that is what the laws of probability predict when diffusion alone is operative. Thus the second law predicts that natural (unintelligent) causes will not do macroscopically describable things which are extremely improbable from the microscopic point of view. The reason natural forces can turn a computer or a spaceship into rubble and not vice versa is probability: of all the possible arrangements atoms could take, only a very small

<sup>3</sup>Dixon has a section "The Entropy Inequality for Open Systems," which contains the inequality, written out in words: "rate of change of entropy inside > rate of entropy flow in — rate of entropy flow out." In any case, even if one refuses to recognize that the boundary integral in (5) represents the (net) rate that entropy is exported, the tautology given in the next section is still illustrated by this application, because this boundary integral still represents the "something" that is crossing the boundary that makes the decrease in entropy not extremely improbable.

<sup>4</sup>In *Classical and Modern Physics*, Kenneth Ford [8] writes "There are a variety of ways in which the second law of thermodynamics can be stated, and we have encountered two of them so far: (1) For an isolated system, the direction of spontaneous change is from an arrangement of lesser probability to an arrangement of greater probability. (2) For an isolated system, the direction of spontaneous change is from order to disorder."

percentage could add, subtract, multiply and divide real numbers, or fly astronauts to the moon and back safely.

Of course, we must be careful to define “extremely improbable” events to be events of probability less than some very small threshold: if we define events of probability less than 1% to be extremely improbable, then obviously natural causes *can* do extremely improbable things.<sup>5</sup> But after we define a sufficiently low threshold, everyone seems to agree that “natural forces will rearrange atoms into digital computers” is a macroscopically describable event that is still extremely improbable from the microscopic point of view, and thus forbidden by the second law — at least if this happens in an isolated system. But it is not true that the laws of probability only apply to isolated systems: if a system is not isolated, you just have to take into account what is crossing the boundary when deciding what is extremely improbable and what is not. What happens in an isolated system depends on the initial conditions; what happens in a non-isolated system depends on the boundary conditions as well.

The “compensation” counter-argument was produced by people who generalized the model equation for isolated systems, but forgot to generalize the equation for non-isolated systems. Both equations are only valid for our simple models, where it is assumed that only heat conduction or diffusion is going on; naturally in more complex situations, the laws of probability do not make such simple predictions. Nevertheless, in “Can ANYTHING Happen in an Open System?” [7], I generalized the equations for non-isolated systems to the following tautology, which is valid in all situations:

If an increase in order is extremely improbable when a system is closed, it is still extremely improbable when the system is open, unless something is entering which makes it not extremely improbable.

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<sup>5</sup>If we repeat an experiment  $2^k$  times, and define an event to be “simply describable” (macroscopically describable) if it can be described in  $m$  or fewer bits (so that there are  $2^m$  or fewer such events), and “extremely improbable” when it has probability  $1/2^n$  or less, then the probability that *any* extremely improbable, simply describable event will *ever* occur is less than  $2^{k+m}/2^n$ . Thus we just have to make sure to choose  $n$  to be much larger than  $k + m$ . If we flip a billion fair coins, any outcome we get can be said to be extremely improbable, but we only have cause for astonishment if something extremely improbable and simply describable happens, such as “all heads,” or “every third coin is tails,” or “only every third coin is tails.” Since there are  $10^{23}$  molecules in a mole of anything, for practical purposes anything that can be described without resorting to an atom-by-atom accounting (or coin-by-coin accounting, if there are enough coins) can be considered “macroscopically” describable.

The fact that order is disappearing in the next room does not make it any easier for computers to appear in our room — unless this order is disappearing *into* our room, and then only if it is a type of order that makes the appearance of computers not extremely improbable, for example, computers. Importing thermal order into a system may make the temperature distribution less random, and importing carbon order may make the carbon distribution less random, but neither makes the formation of computers more probable.

My conclusion, from “Can ANYTHING Happen in an Open System?” [7] is the following:

Order can increase in an open system, not because the laws of probability are suspended when the door is open, but simply because order may walk in through the door.... If we found evidence that DNA, auto parts, computer chips, and books entered through the Earth’s atmosphere at some time in the past, then perhaps the appearance of humans, cars, computers, and encyclopedias on a previously barren planet could be explained without postulating a violation of the second law here.... But if all we see entering is radiation and meteorite fragments, it seems clear that what is entering through the boundary cannot explain the increase in order observed here.

#### 4. The Common Sense Law of Physics

I was discussing the second law argument with a friend recently, and mentioned that the second law has been called the “common sense law of physics.” The next morning he wrote:

Yesterday I spoke with my wife about these questions. She immediately grasped that chaos results in the long term if she would stop caring for her home.

I replied:

Tell your wife she has made a perfectly valid application of the second law of thermodynamics.<sup>6</sup> In fact, let’s take her application a bit further. Suppose you and

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<sup>6</sup>Isaac Asimov [3] writes, “We have to work hard to straighten a room, but left to itself, it becomes a mess again very quickly and very easily.... How difficult to maintain houses, and machinery, and our own bodies in perfect working order; how easy to let them deteriorate. In fact, all we have to do is nothing, and everything deteriorates, collapses, breaks down, wears out — all by itself — and that is what the second law is all about.”



your wife go for a vacation, leaving a dog, a cat and a parakeet loose in the house (I put the animals there to cause the entropy to increase more rapidly, otherwise you might have to take a much longer vacation to see the same effect). When you come back, you will not be surprised to see chaos in the house. But tell her some scientists say, “but if you leave the door open while on vacation, your house becomes an open system, and the second law does not apply to open systems... you may find everything in better condition than when you left.” I’ll bet she will say, “If a maid enters through the door and cleans the house, maybe, but if all that enters is sunlight, wind and other animals, probably not.”

Imagine trying to tell my friend’s wife that, provided her house is an open system, the fact that chaos is increasing in the rest of the universe — or on the sun, provided sunlight enters through the door — means that chaos could decrease in her house while she is gone. Even if the door is left open, it is still extremely improbable that order in the house will improve, unless something enters that makes this not extremely improbable — for example, new furniture or an intelligent human.

Suppose we take a video of a tornado sweeping through a town, and run the video backward. Would we argue that although a tornado turning rubble into houses and cars represents a decrease in entropy, tornados derive their energy from the sun, and the increase in entropy outside the Earth more than compensates the decrease seen in the video, so there is no conflict with the second law? Or would we argue that what we were seeing was too difficult to quantify, so we can’t be sure there is a problem? Some things are obvious even if they are difficult to quantify.

In *Signature in the Cell* [10], Stephen Meyer appeals to common sense in applying the second law to information:

[M]ost of us know from our ordinary experience that information typically degrades over time unless intelligent agents generate (or regenerate) it. The sands of time have erased some inscriptions on Egyptian monuments. The leak in the attic roof smudged the ink in the stack of old newspapers, making some illegible.... Common experience confirms this general trend — and so do prebiotic simulation experiments and origin-of-life research.

A recent article by Andy McIntosh [11] in *International Journal of Design & Nature and Ecodynamics* takes a detailed and technical look at the relationship between entropy and biological information, but also includes appeals to common sense such as:

Both Styer [5] and Bunn [12] calculate by slightly different routes a statistical upper bound on the total entropy reduction necessary to ‘achieve’ life on earth...

these authors are making the same assumption — viz. that all one needs is sufficient energy flow into a [non-isolated] system and this will be the means of increasing the probability of life developing in complexity and new machinery evolving. But as stated earlier this begs the question of *how* a local system can possibly reduce the entropy without existing machinery to do this... machines need to be pre-existing to enable an increase in order and complexity to take place.

## 5. Conclusions

Of course, one can still argue that the spectacular increase in order seen on Earth is consistent with the underlying principle behind the second law because what has happened here is not really extremely improbable. One can still argue that once upon a time, on a special planet called Earth, a collection of atoms formed by pure chance that was able to duplicate itself, and these complex collections of atoms were able to pass their complex structures on to their descendants generation after generation, even correcting errors. One can still argue that, after a long time, the accumulation of genetic accidents resulted in greater and greater information content in the DNA of these more and more complex collections of atoms, and eventually something called “intelligence” allowed some of these collections of atoms to design cars and trucks and spaceships and nuclear power plants. One can still argue that it only seems extremely improbable, but really isn’t, that under the right conditions, the influx of stellar energy into a planet could cause atoms to rearrange themselves into computers and laser printers and the Internet.<sup>7</sup>

But one would think that at least this would be considered an open question, and those who argue that it really *is* extremely improbable, and thus contrary to the basic principle underlying the second law of thermodynamics, would be given a measure of respect, and taken seriously by their colleagues, but we aren’t.

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<sup>7</sup>For further development of the ideas in this paper, see Chapter 5 of my book *In the Beginning and Other Essays on Intelligent Design*, Discovery Institute Press, 2010. ([www.discoveryinstitutepress.com/sewell](http://www.discoveryinstitutepress.com/sewell)).

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